DETAILS EXPLANATIONS

[PART : A]

1. We know that

$$Q = CV$$

and $$i = \frac{dQ}{dt}$$

So, that $$i = \frac{d(CV)}{dt} = C\frac{dV}{dt} \quad ...(1)$$

The power absorbed by the capcitor is given by

$$P = V.i = VC\frac{dV}{dt}$$

and the energy stored by the capcitor is

$$W = \int_0^t p.dt = \int_0^t VC\frac{dV}{dt}.dt$$

$$= \frac{1}{2}CV^2 \quad \text{H. P.}$$

2. V = Drop across C_1 + Drop across C_2

$$\frac{1}{C_{eq}}\int i\,dt = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt$$

or $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

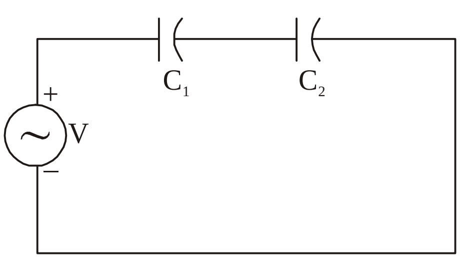
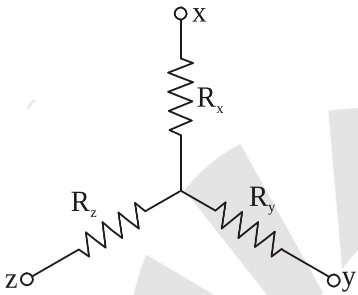
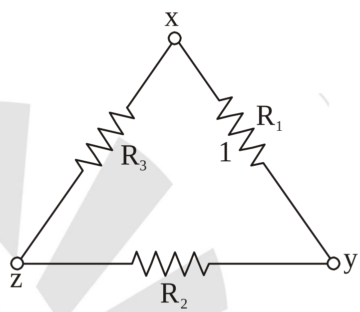


Figure: Series Connection of Capacitance

3.
$$R_x = \frac{R_1R_3}{R_1+R_2+R_3}$$
$$R_y = \frac{R_1R_2}{R_1+R_2+R_3}$$
$$R_z = \frac{R_2R_3}{R_1+R_2+R_3}$$



(a) Star Connection



(b) Delta Connection

4. In a series resonating circuit, Q factor (Quality factor) is define as the ratio of the voltage across the inductor or capcitor to the applied voltage

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

$$Q = \frac{V_L}{V} = \frac{I_o x_L}{I_o R} = \frac{x_L}{R} = \frac{w_o L}{R} \quad \text{(for the inductor)}$$

$$Q = \frac{V_c}{V} = \frac{I_o x_C}{I_o R} = \frac{x_C}{R} = \frac{1}{w_o RC} \quad \text{(For the capcitor)}$$

5. If the mutual flux helps the individual flux, then

$$L_{eq} = \frac{L_1L_2 - m^2}{L_1 + L_2 - 2m}$$

If the mutual flux opposes the individual flux, then

$$L_{eq} = \frac{L_1L_2 - m^2}{L_1 + L_2 + 2m}$$

6. Statement of compensation theorem :

In a linear time-invariant network when the resistance(R) of an uncoupled branch, carrying a current (I), is changed by (ΔR), the currents in all the branches would change and can be obtained by assuming that on ideal voltage source of (V_C) has been connected [Such that V_C = I(ΔR)] in series with (R + ΔR) when all other sources in the network are replaced by their internal resistance.

7. $V_1 = Z_{11}i_1 + Z_{12}i_2$ ...(1)

$V_2 = Z_{21}i_1 + Z_{22}i_2$ ...(2)

From equation (2)

$$i_1 = \left(\frac{V_2}{Z_{21}}\right) - \left(\frac{Z_{22}}{Z_{21}}\right)$$

$$i_2 = \left(\frac{1}{Z_{21}}\right)V_2 + \left(\frac{Z_{22}}{Z_{21}}\right)(-i_2)$$

$$i_1 = CV_2 + D(-i_2)$$

$$C = \frac{1}{Z_{21}};\ D = \frac{Z_{22}}{Z_{21}}$$

8. If the original time function is shifted to the right by a unit of time, its laplace transform is equal to the original laplace transform multiplied by e^{-as}.

i.e., $$L[f(t-a)] = e^{-as}\int_0^\infty f(t)e^{-st}\,dt$$

9. The number of twigs on a tree is always one less than the number of nodes.

Let N = Number of nodes, then number of twigs will be (N – 1) Also, if L represents the total number of links while B the total number of branches, then

$$L = B - (N-1) = B - N + 1$$

This total number of links represent the number of kVL equations.

10. Unit parabola $\left(\frac{t}{2}\right)$ input $R(s) = \frac{1}{s^3}$

$$e(s) = \frac{R(s)}{1+G(s)H(s)}$$

using final value theorem

$$\lim_{s\to 0} e_{ss} = \frac{s.R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0}\frac{s}{s^3(1+G(s)H(s))}$$

$$e_{ss} = \lim_{s\to 0}\frac{1}{s^2(1+G(s)H(s))} = \frac{1}{k_a}$$

Where, $K_a = \lim_{s\to 0} s^2G(s)H(s)$ = Acceleration error constant.

11. If charastics equation consist of either only even powers or only odd powers then closed loop poles of the system lie on jω axis. If these poles are non-repeated, then the system is marginally stable and if they are repeated then the system is unstable.

If there is one or more terms missing in characstics equation system is unstable (provided equation consist of both even or odd powers)

12. Let positive real function

$$f(s) = \frac{P(s)}{Q(s)}$$

Properties

- Both P(s) and Q(s) should be hurtwitz polynomial.
- All poles and zeros of function must lie in left half of s plane.
- The imaginary poles and zero should be simple.
- If f(s) is positive real then $\frac{1}{f(s)}$ also positive real.
- The sum of two positive real function is also positive real but difference is may or may not be positive real.

13. Properties of resonance of parallel LRC circuit is given below:

- Power factor is unity.
- Current at resonance is $\frac{V}{(L/CR)}$ and is in phase with the applied voltage. The value of current at resonance is minimum.
- Net impedance at resonance of the parallel circuit is maximum and equal to $\left(\frac{L}{CR}\right)\Omega$.
- The admittance is minimum and the net susceptance is zero at resonance.
- The resonance frequency of this circuit is given by

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

14. ***The initial value theorem :***

States that if f(t) and it's derivative f'(t) are laplace transformable, then the f(0) of the function f(t) is given by.

$$f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} s f(s)$$

The final value theorem :

States that if the function f(t) and it's derivative f'(t) are laplace transformable, then the final value f(∞) of the function f(t) is given by

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} s f(s)$$

15. Reactive power is the resultant power in watts of an AC circuit when the current wave form is out of phase with the wave from of the voltage, usually by 90^o if the load is purely reactive and is the result of either capacitive or inductive load only when current is in phase with voltage is there actual work done. Such as resistive load an example is powering an incandescent light bulb; in a reactive load energy flow towards the load half the time.

16. It is defined as the fraction of total flax that links the cois. i.e., k the coefficient of cupling

$$= \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

Since, $\phi_{12} < \phi_1$ and $\phi_{21} < \phi_2$ hence the maximum value of k is unity.

$$m = N_2\frac{d\phi_{12}}{di_1} = N_1\frac{d\phi_{21}}{di_2}$$

$$\Rightarrow \quad m^2 = N_1N_2\frac{\phi_{12}\phi_{21}}{di_i di_2}$$

$$m^2 = N_1N_2\frac{k\phi_1.k\phi_2}{i_1i_2} = k^2N_1\frac{\phi_1}{L_1}.N_2\frac{\phi_2}{L_2}$$

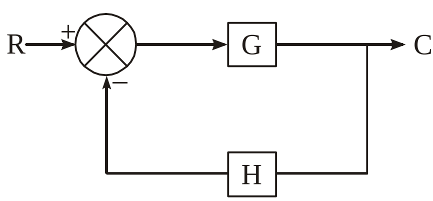
$$m^2 = k^2L_1L_2$$

$$\Rightarrow \quad m = k\sqrt{L_1L_2} \qquad \left[\because L = \frac{N\phi}{i}\right]$$

17. Sensitivity of T w.r.t. $H\left(S_H^T\right)$

$$S_H^T = \frac{\partial T/T}{\partial H/H} = \frac{\partial T}{\partial H}.\frac{H}{T}$$

$$T = \frac{C}{R} = \frac{G}{1+GH}$$



$$\frac{\partial T}{\partial H} = -\frac{G.G.}{(HGH)^2}$$

$$S_H^T = -\frac{G^2}{(1+GH)^2} \times \frac{H(1+GH)}{G}$$

$$= -\frac{GH}{1+GH}$$

Generally GH >> 1

$\therefore$ 1+GH $\simeq$ GH, So that $S_H^T = -1$

18. ***Rise time (t_r) :***

It is the time needed for the response to reach the output from 0% to 100% of steady state value in the first attempt for under damped system and from 10% to 90% of steady state value for over damped system.

$$t_r = \frac{\pi - \phi}{\omega_d} \text{ sec}$$

where $\phi = \tan \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

or $\phi = \cos^{-1}\xi$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

19.
- It adds an open loop pole and zero with pole having dominant effect.
- It improves steady state response and reduce steady state error.
- It reduce the stability of the system.
- Bandwidth of the system decrease.
- Noise in the system reduce therefore S/N ratio at output improves.
- Phase margine is reduced and it's behave like L.P.F.
- Type-2 or higher type system become ustable when lag compensator is used.

20. $\dot{X} = AX + BU$ ..(1)

$y = CX + DU$ ...(2)

taking place of equation (1) we have

$\Rightarrow$ $SX_{(s)} - X(0) = AX_{(s)} + BU_{(s)}$

For transfer function X(0) = 0

$\Rightarrow$ $[SI-A]X_{(s)} = X(0) + BU_{(s)}$

$\Rightarrow \quad X_{(s)} = [SI-A]^{-1}BU_{(s)}$ ...(3)

taking laplace of equation (2) we have,

$Y_{(s)} = CX_{(s)} + DU_{(s)}$ (4)

from equation (3) and (4) we have

$Y_{(s)} = C[SI - A]^{-1}BU_{(s)} + DU_{(s)}$

$$T.F. = \frac{Y_{(s)}}{U_{(s)}} = C[SI - A]^{-1}B + D$$

[PART : B]

21. Here $I_C = \dfrac{V}{x_c}$

and $I_L = \dfrac{V}{z_L} = \dfrac{V}{\sqrt{R^2 + x_L^e}}$

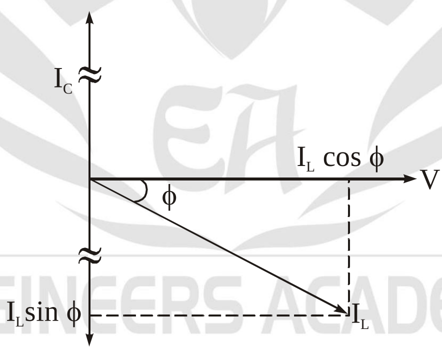
$$= \frac{V}{\sqrt{R^2 + (\omega L)^2}}$$

At resonance

$I_C = I_L \sin\phi$

$z_L = R + jx_L$

$$\frac{V}{x_c} = \frac{V}{z_L} \times \frac{x_L}{z_L} \Rightarrow z_L^2 = x_c x_L$$



Fiagure : Vector diagram of the Circuit

Also $z_L^2 = \dfrac{1}{\omega_0 c} \times \omega_0 L = \dfrac{L}{C}$

i.e. $z_L = \sqrt{\dfrac{L}{C}}$ or $\sqrt{R^2 + (\omega_0 L)^2} = \sqrt{\dfrac{L}{C}}$

$$\Rightarrow \quad R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\Rightarrow \quad \omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow \quad \omega_o^L = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\Rightarrow \quad \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

$$f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

If the resistance of the coil is neglected.

$$\text{Then} \quad f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C}} = \frac{1}{2\pi\sqrt{LC}}$$

22. Mutal inductance between the two coils is defined as the property of the coil due to which it oppose the change of coil is changing the flux sets up in the coil and because of this changing flux emf is induced in the coil called mutually induced emf and the phenomenon is known as mutal inductance.

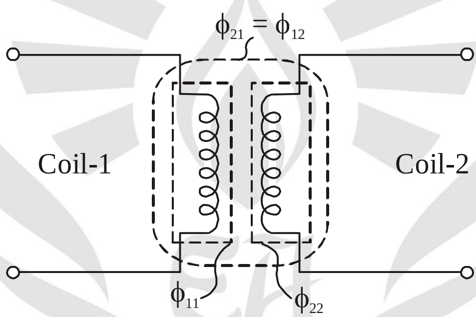


Figure : Linking of Flux

N_1 = Number of turn in coil-1

N_2 = Number of turn in coil-2

$$m\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt}$$

So, that $m = N_2\frac{d\phi_{12}}{di_1}$. Similarly

$$m = N_1\frac{d\phi_{21}}{di_2}$$

23. Maximum power is absorbed by one network from another connected to it at two terminals, when the impedance of one is the complex conjugate of the other.

This mean that for maximum active power to be delivered to the load, the load impedance must correspond to the conjugate of the source impedance (or if the case of direct quantities, be equal to the source impedance).

Proof : Let V be the voltage source R_s the internal resistance of the source and R_L the load resistance.

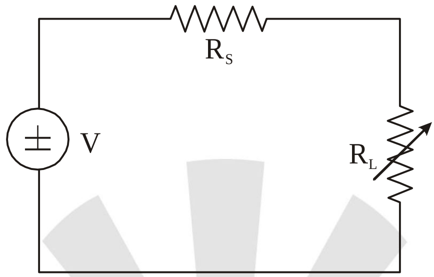


Figure Pusely resistive Circuit with Variable Load Resistance

$\therefore$ Power delivered to the load is

$$P = |I^2|R_L = \frac{V^2 R_L}{(R_s + R_L)^2}$$

For maximum power,

$$\frac{\partial P}{\partial R_L} = V^2\left[\frac{(R_L + R_s) - 2R_L(R_s + R_L)}{(R_s + R_L)^4}\right] = 0$$

$$\therefore \quad R_s = R_L$$

$$P_{max} = \frac{V^2}{4R_L} = \frac{(V/2)^2}{R_L}$$

and thus, the efficiency will be 50%.

24. $Y(s) = Y_1(s) + Y_2(s)$

$$Y(s) = \frac{1}{5s+5} + \frac{1}{2+1/2s} = \frac{1}{5s+5} + \frac{2s}{4s+1}$$

$$Y(s) = \frac{4s+1+10s^2+10s}{5(s+1)(4s+1)} = \frac{10s^2+14s+1}{5(s+1)(4s+1)}$$

$$Y(s) = 0.5\frac{s^2+1.4s+0.1}{(s+1)(s+0.25)}$$

$$I(s) = \frac{0.5(s^2+1.4s+0.1)}{(s+1)^2(s+0.25)} = \frac{A}{s+0.25} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$A = \frac{0.5(0.0625 - 0.35 + 0.1)}{0.5625} = -\frac{1}{6}$$

$$C = \frac{0.5(1-1.4+0.1)}{-0.75} = \frac{1}{5}$$

$$B = 0.5\frac{di}{ds}\left[\frac{s^2+1.4s+0.1}{s+0.25}\right]\Bigg|_{s=-1} = \frac{2}{3}$$

$$i(t) = \left(-\frac{1}{6}e^{-0.25t}+\frac{2}{3}e^{-t}+\frac{1}{5}te^{-t}\right)u(t)$$

25. In a passive AC circuit, let the instantaneous voltage be $V = V_m \sin \omega t$ while the current is given by $i = I_m \sin(\omega t - \phi)$ ϕ is phase difference between the voltage and current at any instant.

$\therefore$ The instantaneous power P is thus given by

$P = V.i = V_m I_m \sin \omega t \sin(\omega t - \phi)$

$= \frac{1}{2} \times 2 V_m I_m \sin \omega t \sin(\omega t - \phi)$

$= \frac{1}{2} V_m I_m [\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)]$

$= \frac{1}{2} V_m I_m [\cos \phi - \cos(2\omega t - \phi)]$

$= \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ ...(1)

In above equation second term avg. value is zero since the avg. of a sinusoilal quantity of double frequency over a complete cycle is zero. Thus the ingtantaneous power consist only the term $\left(\frac{1}{2}V_m I_m \cos\phi\right)$ which is the avg. power in the passive circuit.

26. ***Tree:***

A tree is a set of branches with every node connected to every other node in such a way that removal of any branch destroys this property.

Atternately a tree is defined as a connectes sub-graph of a connected graph containing all the nodes of the graph but not containing any loops. Branch of a tree are called twigs. A tree contains (n – 1) twigs where n is the number of nodes in the graph.

Tree have the following properties :

- There exists only one path between any pair of nodes in a tree.
- A tree contains all nodes of the graph.
- If n is the number of nodes of the graph there are (n –1) branches in the tree.
- Trees does not contain any loops.
- Every connected graph has at least one tree.
- The minimum terminals nodes in a tree are two.

27. ***Symmetrical π-Attenuator :***

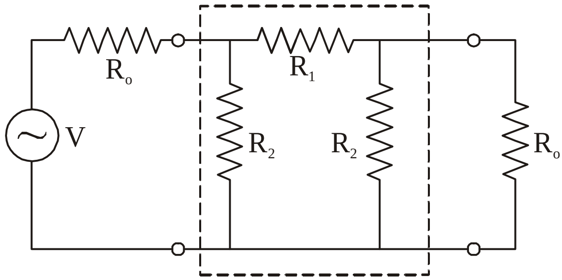


Figure shows a symmetrical π-attenuator which is placed in between source of internal impedance R_o and load R_o hence R_o represents the characteristic or design impedance of the attenuator.

Since $Z_0 = R_o$ and r = α, we have

$$\frac{I_1}{I_2} = e^r = e^\alpha = N$$

For a symmetrical p network the series and shunt arm resistances in terms of R_o and α are given by

$$R_1 = R_o \sin h\, \alpha = R_o\left(\frac{e^\alpha - e^{-\alpha}}{2}\right)$$

$$= R_o \frac{N-(1/N)}{2} = R_o \frac{N^2-1}{2N}$$

and $$R_2 = R_o \coth\left(\frac{\alpha}{2}\right) = R_o\left(\frac{e^{\alpha/2}+e^{-\alpha/2}}{e^{\alpha/2}-e^{-\alpha/2}}\right)$$

$$= R_o\left(\frac{e^\alpha+1}{e^\alpha-1}\right) = R_o \frac{N+1}{N-1}$$

28. $$Y_1(j\omega) = \frac{1}{R_L + j\omega L}$$

$$Y_2(j\omega) = \frac{1}{R_C - (j/\omega c)}$$

$$Y(j\omega) = Y_1(j\omega) + Y_2(j\omega)$$

$$= \frac{1}{R_L + j\omega_L} + \frac{1}{R_C - j/\omega c}$$

At resoance $w = w_0$

$$I_m[Y(j\omega)] = 0$$

$$Y(j\omega) = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j/\omega c}{R_c^2 + 1/\omega^2 C^2}$$

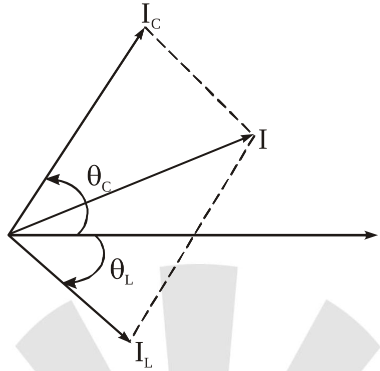


Figure : Phasor diagram

$$I_m[Y(j\omega)] = 0 = \frac{-j\omega_o L}{R_L^2 + \omega_o^2 L^2} + \frac{(j/\omega_o C)}{R_c^2 + \frac{1}{\omega_o^2 C^2}} = 0$$

$$\frac{\omega_o L}{R_L^2 + \omega_o^2 L^2} = \frac{1/\omega_o C}{\frac{\omega_0^2 R_C^2 C^2 + 1}{\omega_0^2 C^2}}$$

$$\Rightarrow \quad \frac{L}{C} = \frac{R_L^2 + \omega^2 L^2}{1 + \omega^2 R_C^2 C^2} \text{ H.P.}$$

29. ***Observability :***

A system is said to be observable, if it is possible to determine the output of the system by simply knawing the internal states of the system at any given point of time kalman test of observability:

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

- A system is completely observable if rank of matrix q_o is equal to the order of the system.
- System will be unobservable if rank of Q_0 is less than the order of the system.
- The number of observable state are equal to rank of matrix Q_0.
- The number of unobservable states are equal to (n – r).
  Where n = Order of the system.
  and r = Rnak of the system matrix Q_0.

30. In unbalance boding, due to flow of unequal current in each of the phases, a voltage appears at the neutral and its value can be determined as follows:

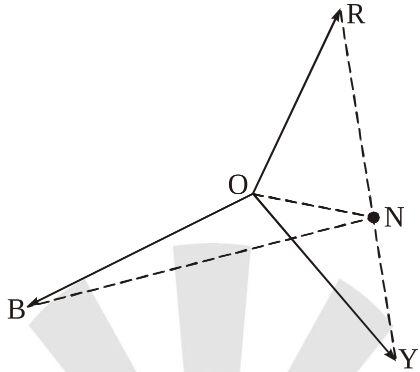


Figure : Neutral Voltage appearing

$I_R + I_Y + I_B = V_{RO}Y_R + V_{yo}Y_y + V_{BO}Y_B$

$= (Y_{RN}–V_{ON})Y_R+(V_{yN}–V_{ON})Y_y+(V_{BN}–V_{ON})Y_B$

$= (Y_{RN}Y_R+V_{yN}Y_y+V_{BN}Y_B)–V_{ON}(Y_R+Y_y+Y_B)$

However, as per kCL, $I_R + I_y + I_B = 0$

Thus,

$V_{RN}Y_R+V_{YN}Y_y+V_{BN}Y_B–Y_{ON}(Y_R+Y_y+Y_B) = 0$

or $\quad Y_{ON} = \dfrac{V_{RN}Y_R + V_{yN}Y_y + V_{BN}Y_B}{Y_R + Y_y + Y_B}$

where, V_{ON} = Represents the neutral shift.

31. It is used to improve both transient as well as steady state response of the system.

Transfer function

$$G(s) = \underbrace{\left(\frac{s+z_{c1}}{s+P_{c1}}\right)}_{\text{Lead}}\underbrace{\left(\frac{s+z_{c2}}{s+p_{c2}}\right)}_{\text{Lag}} \quad |z_{c1}| < |p_{c1}| \text{ and } |z_{c2}| > |p_{c2}|$$

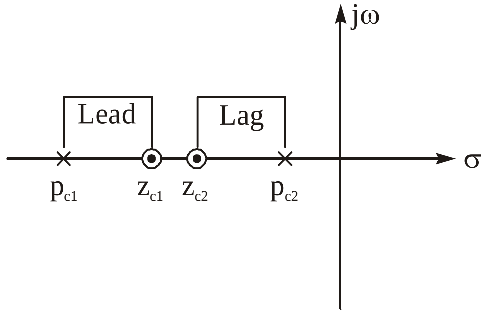
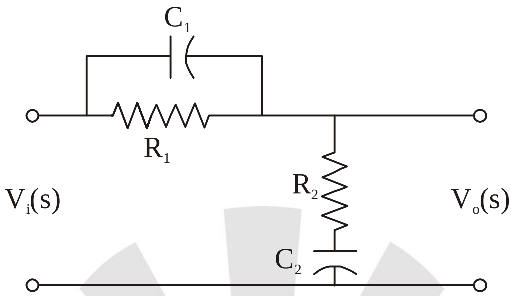


Figure : Pole zero Diagram

$$G(s) = \underbrace{\left(\frac{1+s\tau_1}{1+s\tau_1\alpha}\right)}_{\text{Lead}}\underbrace{\left(\frac{1+s\tau_2}{1+s\tau_2\beta}\right)}_{\text{Lag}}$$

Practical implementation of lag-lead compensator:



32. ***Controllability :***

At system is said to be controllable, if it is possible to transfer the system from one state to another state by using input vector over a specified period of time kalman test of controllability.

Controllability matrix

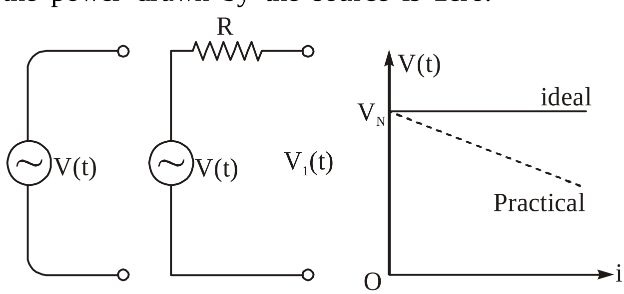
$$Q_c = [B : AB : A^2B...A^{n-1}B]$$

- A system will be controllable if rank of matrix Q_c is equal to order of the system.
- System will be uncontrollable if rank of matrix Q_c is less than the order of the system.
- The number of uncontrollable state are equal to = n – r.
  where, n = Order of the system.
  r = Rank of the matrix Q_c.
- Number of controllable states are equal to r(rank.)

[PART : C]

33. ***An ideal voltage source has the following characteristics :***

- It is a voltage generator whose output voltage remains absolutely constant whatever be the value of the output current.
- It has zero internal resistance so that voltage drop in the source is zero.
- the power drawn by the source is zero.



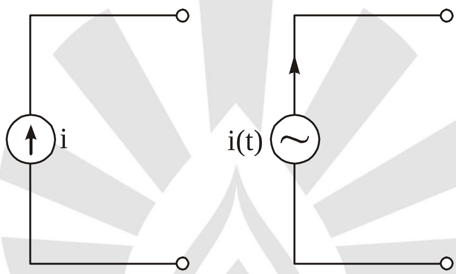
In practical voltage source, the voltage does not remain constant, but falls slightly; this is taken care of by connecting a small resistance(R) in series with the ideal source in this case, the terminal voltage will be

$$V_1(t) = V(t) - iR$$

i.e., it will decreases with increase in current i.

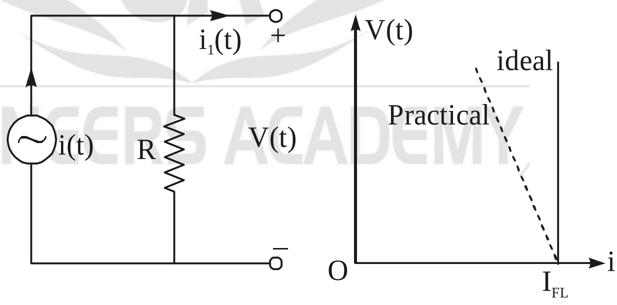
An Ideal Current Source has the following Characteristics :

- It produces a constant current irrespective of the value of the votlage across it.
- It has infinity resistance.
- It is capable of supplying infinity power.



In practical current sources, the output current does not remain constant but decreases with increase in voltage. So, a practical current source is represented by an ideal current source in parallel with a high resistance(R) and the output current becomes.

$$i_1(t) = i(t) - \frac{V(t)}{R}$$



Loading of Sources :

It has been mentioned that the output voltage of a voltage source decreases as the load current increases. If the sources is loaded in such away that the output (or load) voltage faly below a specified full bad value, then the source is said to be loaded and the situation is known as loading of source.

For example, we consider a voltage source of 100 V as shown in figure with an internal resistance of 1 Ω.

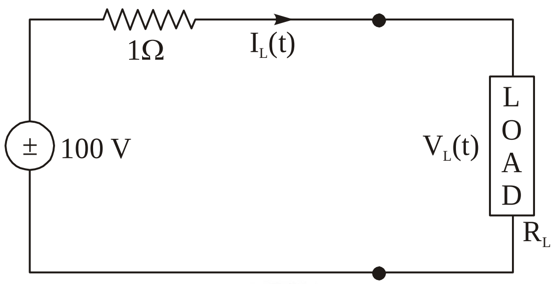


Figure Loading of Source

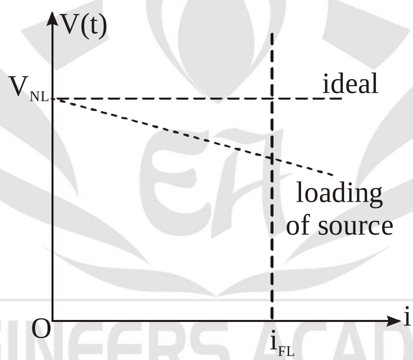
Here, load current,

$$i_L(t) = \frac{100}{R_L + 1}$$

No load voltage is

$$V_{NL} = 100 \text{ V}$$

If the specified full load current is 10 A then the load resistance on full load is



$$R_L = \left(\frac{100}{10} - 1\right) = 9\ \Omega$$

Then the full load voltage is

$$V_{FL} = I_{FL} \times R_L$$

$$= 10 \times \left(\frac{100}{10} - 1\right) = 90 \text{ V}$$

If the load resistance is increases beyond 9 W, the load voltage falls below the specified full load voltage of 90 V in that case, the source is said to be loaded.

34. (i) The equation relating inductance and flux linkages can be rearranged as follows :

$$\lambda = L_i$$

Taking the time derivative of both sides of the equation yields

$$\frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dt}$$

in most physical cases, the inductance is constant with time and so

$$\frac{d\lambda}{dt} = L\frac{di}{dt}$$

By faraday's law of induction we have

$$\frac{d\lambda}{dt} = -E = V$$

Where E is the electromotive force (emf) and V is the induced voltage. Note that the emf is opposite to the induced voltage. Thus

$$V = L\frac{di}{dt} \quad ...(1)$$

$$i(t) = \frac{1}{L}\int_0^t V(t)\,dt + i(0) \quad ...(2)$$

Where i(t) is the initial current. WHen initial current is zero,

$$i(t) = \frac{1}{L}\int_0^t V(t)\,dt$$

These equation together state for a steady applied voltage V, the current changes in a linear manner at a rate proportioal to the applied voltage, but inversely proportional to the inductance. Conversely, if the current through the inductor is changing at a constant rate, the induced voltage is constant.

From the equation (1) it is clear that for a abrupt change in current, the voltage across the inductor becomes infinite, Also, from equation (2) it is observed that for a finite change in voltage in zero time, the integral must be zero.

therefore, the current through an inductor cannot change instantaneously.

(ii) The relation between charge and voltage in a capacitor is written as

$$Q = CV$$

and the current

$$i = \frac{dQ}{dt}$$

$$i = C\frac{dV}{dt} + V\frac{dC}{dt}$$

in most physical cases, the capacitance is constant with time

$$i = C\frac{dV}{dt} \qquad ...(1)$$

$$\Rightarrow \qquad dV = \frac{1}{C} i dt$$

Taking integration on both side

$$\int_0^{V_c} dV = \frac{1}{C}\int_0^t i\,dt$$

or $$V_C(t) = \frac{1}{C}\int_0^t i\,dt + V_C(0)$$

Where, $V_C(0)$ is the initial voltage across the capacitor for zero initial voltage.

$$V_C = \frac{1}{C}\int_0^t i\,dt \qquad ...(2)$$

From equation (1) it is clear that for an abrupt change of voltage across capacitor, the current becomes infinite. Also, from the equation (2) it is observed that for a finite change of current in zero time, the integral must be zero. There for, the voltage across a capacitor connot change instantangeously.

35. The time response of an underdamped control system exhibits damped oscillation prior to reaching steady state the specifications pertaining to time response during transient part are shown in figure and explain below.

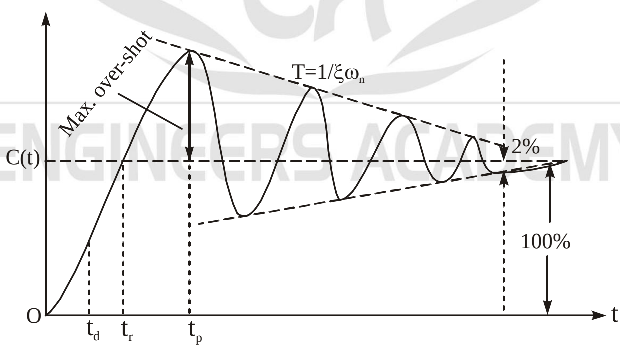


Figure : Transient Response Specifications

- ***The rise Time (t_r)***

The rise time is the time needed for the response to reach from 10% to 90% or 0 to 100% of the desired value of the output at the very first instast usually 0 - 100% basis is used for underdamped systems and 10% to 90% for overdamped system.

The rise time (0 to 100%) for underdamped second order control system is given below.

$$t_r \text{ (rise time)} = \frac{\pi - \phi}{\omega_d} \text{sec}$$

where $\phi = \cos^{-1}\xi$

or $\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

so $$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_n\sqrt{1-\xi^2}}$$

and $\omega_d = \omega_n\sqrt{1-\xi^2}$

- ***Delay Time (t_d)***

  It is the time taken by the response to reach 50% of the steady state value.

$$t_d = \frac{1+0.7\xi}{\omega_n} \text{ sec}$$

- ***Settling Time (t_s)***

  For an underdamped system the magnitude of the oscillation present in the output time response decay exponentially with a time constant $\frac{1}{\xi\omega_n}$. The time needed to settle down aforesaid oscillations within 2% of desired value of the output is known as settling time and denoted as t_s. The setting time for a second order control system is approximately for times the time constant of the system hence.

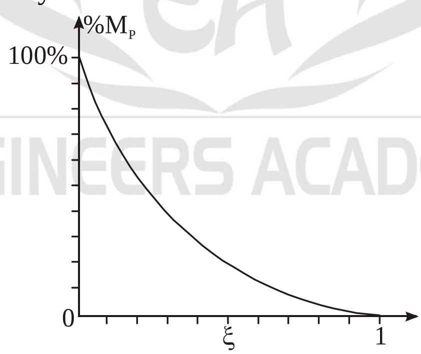


Figure : Graph between M_p and ξ

$$t_s = \frac{4}{\xi\omega_n}\text{; for 2\% tolerence}$$

$$t_s = \frac{3}{\xi\omega_n}\text{ ; for 5\% tolerence}$$

- ***Peak Overshoot (M_p)***

  It is the percentage overshoot above steady state value in first attempt.

$$\%M_P = \frac{C_p - C_\infty}{C_\infty} \times 100$$

where, C_p = Peak value of output.

and C_∞ = Steady state output.

Peak over shoot (M_p) in terms of damping ratio.

$$\%M_p = 100\, e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

Note : For n[th] peak overshoat

$$\%M_{pn} = 100\, e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

where, n = 1, 3, 5,...; for over shoots

n = 2, 4, 6,...; for under shoots

- ***Peak time (t_p)***

  It is the time needed for the response to reach at its first peak (maximum) over shoot (M_p)

$$t_p = \frac{\pi}{\omega_d} \text{ ; for first peak}$$

where, n = 1, 3, 5,...,for over shoots.

n = 2, 4, 6,...,for under shoots.

$$t_{pn} = \frac{n\pi}{\omega_d} \text{; for n}^{th} \text{ Peak}$$

and $\omega_d = \omega_n\sqrt{1-\xi^2}$

36. $$\omega_o = \sqrt{\frac{1}{Lc} - \left(\frac{R}{L}\right)^2}$$

$$LC = 10.5 \times 10^{-3} \times 105 \times 10^{-9} = 1102.5 \times 10^{-12}$$

$$\frac{1}{LC} = 9.07 \times 10^{-4} \times 10^{12} = 9.07 \times 10^8 = 907 \times 10^6$$

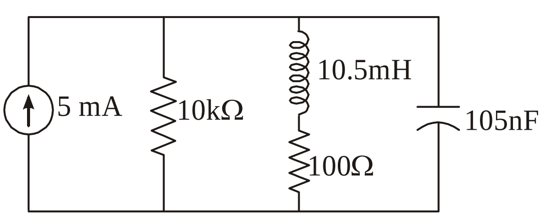


Figure (a)

and $\frac{R}{L} = \frac{100}{10.5\times10^{-3}} = 9.52 \times 10^3$

and $\left(\frac{R}{L}\right)^2 = (9.52)^2\times(10^3)^2 = 90.63 \times 10^6$

$$\omega_o = \sqrt{907\times10^6 - 90.63\times10^6}$$
$$= \sqrt{816.37\times10^6}$$
$$= 28.57 \times 10^3 \text{ r/s}$$

(i) $f_o = \frac{\omega_o}{2\pi} = \frac{28.57\times10^3}{2\pi} = 4.547$ KHz

(ii) Q of the coil, $Q_c = \frac{\omega_o L}{R}$

$$= \frac{28.57\times10^3\times10.5\times10^{-3}}{100} = 3$$

(iii) Bandwidth of entire circuit

$$\frac{\omega_o}{Q} = \frac{28.57\times10^3}{3}$$

The coil shown in figure (a) is transformed to its parallel equivalent as shown below in figure (b).

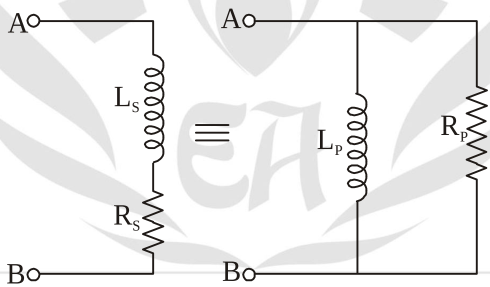


Figure (b)

$$L_p = L_s\left(1+\frac{1}{Q_c^2}\right)$$

$$R_p = R_s\left(1+Q_c^2\right)$$

where, Q_c = Q of the coil $= \frac{\omega_o L_s}{R_s} = \frac{R_p}{\omega_o L_p} = 3$

Using formula L_p and R_p

$$R_p = 10\ \Omega \times R_s = 10 \times 100 = 1000\ \Omega$$

$$L_p = \frac{10}{9}\times L_s = \frac{10\times10.5}{9} = \frac{105}{9} = \frac{35}{3}\text{mH}$$

∴ The circuit in figure in transformed to equivalent parallel resonant circuit shown in figure.

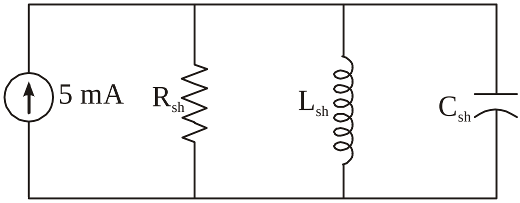


Figure (c)

where $R_{sh} = R_p \parallel 10\ k\Omega$

$$R_{sh} = \frac{R_p \times 10^4}{R_p + 10^4} = \frac{10^3 \times 10^4}{10^3 + 10^4} = \frac{(10)^4}{11}\ \Omega$$

$$L_{sh} = \frac{35}{3}\ \text{mH and } C_{sh} = 105\ \text{nF}$$

Q of the entire circuit

$$Q_e = \frac{R_{sh}}{\omega_o L_{sh}} = \frac{10^4 \times 3}{11 \times 28.57 \times 10^3 \times 35 \times 10^{-3}}$$

$$= 2.73$$

Bandwidth of entire circuit

$$\frac{f_o}{Q_e} = \frac{4.547 \times 10^3}{2.73} = 1665.6\ \text{Hz}$$

(iv) From figure (a) voltage across coil = Voltage across 'C' = Coltage across 10 kΩ

at resonacne $\omega = \omega_0$, resonance impedance = R_{sh} across 5 mA

∴ Voltage across coil at resonance

$$= 5\ \text{mA} \times \frac{10}{11}\text{k}\Omega = 4.5\ \text{V}$$

(v) Current through the capacitor at resonance

$$I_c = Q_e I_{source}$$
$$= 2.73 \times 5\ \text{mA} = 13.65\ \text{mA}$$

37. The cut set matrix provides a compact and effective means of writing agebraic equcation given branch voltages in terms of tree branches. The number of independent node-pair terminals is equal to the number of tree branches. It is possible to express the potential difference in terms of tree branch voltage.

Procedure of forming fundamental cut-set matrix (Q)

Step-I : Arbitrarily a tree is selected in the graph.

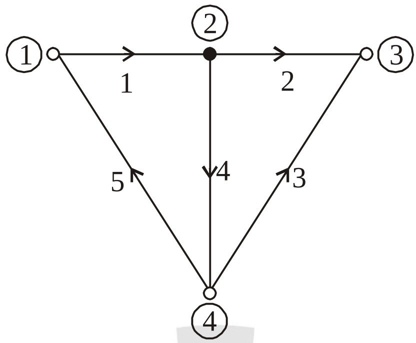


Figure (a)
An oriented Graph

Step-II : Form fundamental cu-sets with each twig in the graph for the entire tree.

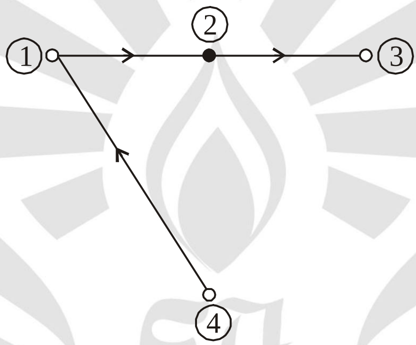


Figure (b)
Selected Tree

Step-III : Assume directions of the cut-sets oriented in the same direction of the concerned twig.

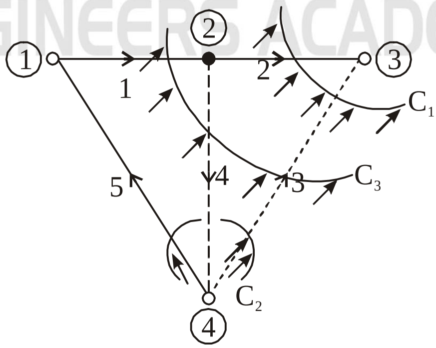


Figure (c)
Cut-Sets in Selected Tree

Step-IV : From fundamental cut set matrix $[Q_{kj}]$
where, Q_{kj} = 1 when branch b_i has same orientation of the cut-set.
q_{kj} = –1, when branch b_i has opposite orientation of the cut-set k.
Q_{kj} = 0 when b_j is not in the cut-set k.

An illustration :

An oriented graph is shown in figure (a) let us first select a tree arbitrarily figure (a) next, cut sets are formed in figure (b). Each cut set has only one twig.

Cut-set C_1 : Twig 2, link 3.

Cut-set C_2 : Twig 5, link 3 and 4 cut set

Cut-set C_3 : twig 1, link 4 and 3.

In may be noted that for cut-set C_1, Q_{11}(i.e. the branch-1 linking cut-set C_1) has cofficient 0 since branch 1 does not link C_1. On the other hand, branch 2 has same orientation of Cut-Set C_2 and hence Q_{12} = +1. Link 3 has also same direction like C_1 and hence Q_{13} = +1. Branches 4 and 5 do not link C_1 and hence Q_{14} and Q_{15} each has zero coefficient. Cut-set 2 links twig 5 whose direction is the same as C_2. Thus Q_{25} = +1. Similar reasonings hold for link 3 and hence Q_{23} is +1, but link 4 has direction opposite to that of the cut-set 2. Thus Q_{24} = –1.

Since branches 1 and 2 do not link C_2 hence Q_{21} and Q_{22} are O each cut set 3 links twig 1 and links 4 and 3 observing the direction of these branches we can write Q_{31} = +1, Q_{34} = –1 and Q_{33} = 1. Since twig 2 and 5 do not link C_3 hence Q_{32} = 0, Q_{35} = 0. Thus we can now form the fundamental cut-set matrix as shown below.

Cut Sets ↓ Branches →

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & +1 & +1 & 0 & 0 \\ 0 & 0 & +1 & -1 & +1 \\ +1 & 0 & +1 & -1 & 0 \end{bmatrix} \end{array}$$

The current balance equation can be obtained form the cut-set matrix as follows:

$$i_2 + i_3 = 0$$
$$i_3 - i_4 + i_5 = 0$$
$$i_3 - i_4 = 0$$

Arrow on cut-set indicate the direction of cut-set.

38. ***Procedure for plotting the root locus for a given open-loop transfer function as given below :***

- ***Starting points :***

The roat locus starts (k = 0) from the open-loop poles.

- ***Ending points :***

  The root locus terminates ($k = \infty$) either on open-loop zero or infinity.

- ***Number of Branches :***

  Number of brances of the root locus are

  $$N = P \text{ if } P > Z$$

  $$N = Z \text{ if } Z > P$$

  usually, $P > Z$, therefore, $N = P$.

- ***Existence on real axis :***

  The existence of the root locus on a section of real axis is confirmed if the sum of the open-loop poles and zeros to the right of the section is odd.

- ***Break away point :***

  On the root locus between two open-loop poles the roots nove towards each other as the gain factor K is increased till they are coincident.

  At the coincident point the value of K is maximum as for a the portion of the root locus between the two open-loop poles in concerned. Any further increase in the value of K, breaks the root locus in two parts. The break away points can be determined by rewriting the characteristic equation and therefore solving for the vlaue of 's' from the equation given below:

  $$\frac{dk}{ds} = 0$$

- ***The angle of Asymptotes :***

  For higher value of k the root locus branches are approximated by asymptotic lines. Making an angle with the real axis given by

  $$\frac{(2k+1)\times 180^{o}}{P-Z}$$

  where, $k = 0, 1, 2, \ldots$ upto $(P - Z) - 1$.

- ***Intersection of asymptotes on real axis :***

  The asymptotes intersect at a point x on the real axis given by

  $$x = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{P-Z}$$

- ***Intersection Points with Imaginary Axis :***

  The value of k and the point at which the root locus branch crosses the imaginary axis is determined by applying Routh criterian to the characteristic equation. The roots at the intersection point are imaginary.

- ***The angle of departure from complex pole :***

  Is given by

  $\phi_d = 180^o\ (\phi_P - \phi_Z)$ where

  ϕ_p = is the sum of all the angles subtended by remaining poles.

  and ϕ_z = is the sum of all the angles subtended by zeros.

  The ϕ_d is tangent to the root locus at the complex pole.

- ***The Angle of arrival to complex Zeros :***

  Is given by

  $\phi_a = 180 - (\phi_z - \phi_p)$

  where, ϕ_p = Sum of all the angle subtended by poles.

  The angle of arrival is tangent to the root locus at the complex zero.

  The calculation of the value of k at a specified point on the root locus, determinations of damping ratio ε, assessment of time response and stability etc.

39. A network is termed to be reciprocal if the ratio of the response variable to the excitation variable remains identical even if the positions of the response and excitation in the network are interchanged.

A two port network is said to be symmetrical if the input and output ports can be interchanged without altering the port voltages and currents.

Reciprocity in Z-parameter representation

Let the Z parameter based two port network be represented as shown in figure.

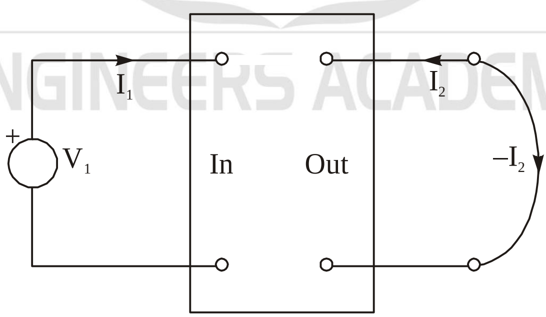


Figure : Two port network representation

Here I_2 is assumed to be in reversed direction of I_2 assumed in two port network for ABCD parameter representation.

Here, from the governing Z-parameter network equcation

$V_1 = Z_{11}I_1 + Z_{12}I_2$

and $V_2 = Z_{21}I_1 + Z_{22}I_2$

We write as

$$V_1 = Z_{11}I_1 - Z_{12}I_2 \quad ...(1)$$

$$0 = Z_{21}I_1 - Z_{22}I_2 \quad ...(2)$$

Here I_2 is –ve as its direction is reversed to the originally assumed direction, V_2 is zero as output is shorted.

From equation (1) and (2)

$$I_2 = \frac{V_1Z_{21}}{Z_{11}Z_{22} - Z_{21}Z_{12}} \quad ...(3)$$

Let us now interchange the input and output voltage and current in figure.

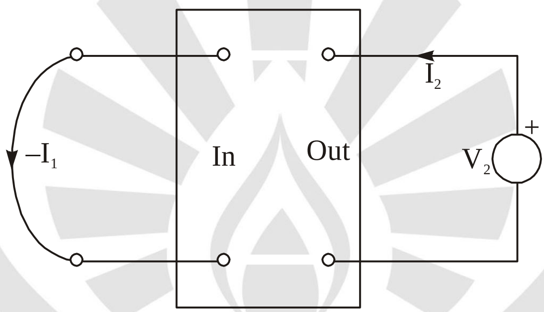


Figure : Voltage and CUrrent are interchanged for the two port network

Here, from the same set of governing equation of Z parameter network,

$$0 = Z_{11}I_1 + Z_{12}I_2 \quad ...(4)$$

and $$V_2 = -Z_{21}I_1 - Z_{22}I_2 \quad ...(5)$$

[Here, I_1 is negative as its direction is opposite to that originally assumed, V_1 is zero as input terminals are shorted]

solving equation (4) and (5)

$$I_1 = \frac{V_2Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \quad ...(6)$$

Assume $V_1 = V_2$, comparision of equation (3) and (6) yields

$$Z_{12} = Z_{21} \quad ...(7)$$

equation (7) represents the condition of reciprocity.

Symmetry in Z-parameter representation :

Applying voltage V at input port and keeping the output port open,

$$V_1 = Z_{11}I_1$$

i.e., $Z_{11} = \left.\frac{V}{I_1}\right|_{I_2=0}$

Again, applying voltage V at output port and keeping open circuit at input port,

$$V = I_2 Z_{22}$$

i.e., $Z_{22} = \left.\frac{V}{I_2}\right|_{I_1=0}$

COndition of symmetry is observed when

$$\left(\frac{V}{I_1}\right) = \left(\frac{V}{I_2}\right)$$

Leading to the condition

$$Z_{11} = Z_{22}$$

❍❍❍